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$$\frac{d^2s}{dt^2} = -S\frac{dr}{ds} - S'\frac{dr'}{ds} \ldots\ldots (1).$$

But $S = m/r$, $S' = m/r'$, and (1) becomes

$$\frac{d^2s}{dt^2} = -\frac{m}{r}\frac{dr}{ds} - \frac{m}{r'}\frac{dr'}{ds} \ldots\ldots (2).$$

Multiply by $2(ds/dt)$ and integrate ; then

$$\frac{ds^2}{dt^2} = -m\log r^2 - m\log r'^2 + C \ldots\ldots (3).$$

When $r=a$, $r'=a$, $\frac{ds}{dt}=\beta$; $\therefore C=\beta^2 - m^2\log\frac{1}{a^4}$, and (3) is

$$\beta^2 = m^2\log\frac{1}{r^2r'^2} + \beta^2 - m^2\log\frac{1}{a^4} \quad \ldots\ldots (4),$$

or, $rr' = a^2 \ldots\ldots (5)$, a lemniscate.

[Other solutions of this problem will appear in the next issue.]

DIOPHANTINE ANALYSIS.

73. Proposed by M. A. GRUBER, A. M., War Department, Washington, D. C.

Find integral values for x and y in $\begin{pmatrix} 2x^2 - y^2 = \square \\ 2y^2 - x^2 = \square \end{pmatrix}$.

I. Solution by the PROPOSER.

$$2x^2 - y^2 = \square = a^2 \ldots\ldots (1), \quad 2y^2 - x = \square = (2).$$

From (1), $y^2 = 2x^2 - a^2$. Substituting this in (2), we have $3x^2 - 2a^2 = b^2$. Whence $x = \frac{1}{3}\sqrt{[3(2a^2 + b^2)]}$, and $y = \frac{1}{3}\sqrt{[3(a^2 + 2b^2)]}$.

As far as I know, the only method of rationalizing both radicals is to put $a=b$. Then $x=y=a=b$.

Accordingly, no *different* integral values can be found for x and y.

This problem is the key to Problem 62, "To find four squares in arithmetical progression."

The roots of the four squares would then be, respectively,

$$a, \quad \tfrac{1}{3}\sqrt{[3(2a^2 + b^2)]}, \quad \tfrac{1}{3}\sqrt{[3(a^2 + 2b^2)]}, \quad b.$$

The common difference of the squares is $\frac{1}{3}(b^2 - a^2)$.

According to the above solution, the roots of the four squares could not *all* be rational integers ; *one* of them, at least, must be a *surd*. It is evident, however, that an infinite number of sets of four squares can be found in which *two* of the roots are rational integers.

Put $a=1$ and $b=2$. Then the roots of the four squares are 1, $\sqrt{2}$, $\sqrt{3}$, 2.

Put $a=1$ and $b=5$. Then the roots are 1, 3, $\sqrt{17}$, 5.

A similar proof was received from *CHARLES C. CROSS.*

II. Solution by A. H. BELL, Hillsboro, Ill.

Take $2y^2-x^2=\square$, or $x^2-2y^2=-\square=-1=-4$, etc.

In $x^2-2y^2=-1$.....(3), the integral values for x and y are the alternate convergent fractions for the $\sqrt{2}$=to

$$x/y=1/1,\ 7/5,\ 41/29,\ \text{etc}.....(4).$$

For the next, $x^2-2y^2=-4$. $(4)\times\sqrt{4}$,

$$\frac{x}{y}=\frac{1\times2}{1\times2},\ \frac{7\times2}{5\times2},\ \frac{41\times2}{29\times2},\ \text{etc.}$$

Consequently the interchangeable values of x and y must be found in the first fraction and no other.

III. Solution by JOSIAH H. DRUMMOND, LL. D., Portland, Me.

$2x^2-y^2=\square$ .....(1), and $2y^2-x^2=\square$ .....(2).

Take $x=my$ and $2m^2-1=\square$ .....(3), and $2-m^2=\square$ .....(4).

Then $m<\sqrt{2}$ and $m>\frac{1}{2}\sqrt{2}$.

It is manifest that both (3) and (4) are rational when $m=1$, which is $<\sqrt{2}$ and $>\frac{1}{2}\sqrt{2}$.

Then in (3) take $m=n+1$, and we have

$$2n^2+4n+1=\square=(\text{say})(qn-1)^2, \text{ whence}$$

$$n=\frac{2(q+2)}{q^2-2} \text{ and } m=n+1=\frac{(q+1)^2+1}{q^2-2}.$$

Substituting this value of m in (4) and reducing by the usual methods, we find $q=0$ and $m=\pm1$.

Hence $x=\pm y$ and the integral values are any equal numbers, positive or negative, or one positive and the other negative.

MISCELLANEOUS.

65. Proposed by J. M. COLAW, A. M., Monterey, Va.

Three circles, radii in ratio 1, 3, 5, are tangent externally and enclose one acre; what are the radii ?